ACCELERATION OF CONDUCTORS TO SUPERSONIC SPEEDS IN A PULSED MAGNETIC FIELD

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The effect of the shape of the current-density curve and the initial angle of acceleration on the velocity of projection of current-carrying conductors in a pulsed magnetic field produced in the discharge of a capacitive energy accumulator is investigated. A variational problem is solved to determine the optimum current-density function in the projected body for a given magnetic-induction function. It is shown that the relation $j = KB$, where K is a constant, gives the maximum velocity. For a uniform magnetic field varying as a damped sinusoid, expressions are obtained for the current density in the accelerated body, the velocity of projection, and the acceleration path. It is shown that there is an optimum initial angle of acceleration depending on the amplitude and frequency of the accelerating force and the acceleration path. A procedure is presented for the approximate design of a hypersonic electromagnetic accelerator. In accord with the conclusions of the theory an experimental arrangement is set up and a study is made of the projection of conductors in a pulsed magnetic field. A maximum velocity of 10.5 km/sec is obtained for 0.16-mm diameter aluminum wires.

1. The devices employed in high-velocity impact studies use electrical energy to accelerate solid bodies [1]. Among the promising devices of this kind are electromagnetic accelerators using the force of a pulsed magnetic field on a current-carrying conductor [2-4]. The velocities obtainable with these devices do not exceed 4-5 km/sec for accelerated bodies having mass of 10^{-6}-10^{-4} kg [2-5]. The reasons for this are the relatively low values of the limiting velocity of the conductors because of heating, and processes at the moving contacts between the accelerated conductor and the guide rails; arcing at the contacts leads to the shunting of the accelerated body by a plasma and a decrease in the velocity attainable.

The limiting velocity set by heating conditions can be increased and the arcing at the moving contacts between the accelerated conductor and the guide rails can be reduced in these devices by the "separation of currents," permitting separate control of the accelerating magnetic field and the current in the conductor. In this case the current density in the projected body can be fixed by the condition for admissible heating, and the current producing the accelerating magnetic field can be chosen in accord with the mechanical strength and the thermal properties of the coil. By decreasing the current in the projected body and decreasing its cross section arcing at the contacts can be reduced while maintaining a large acceleration.

The theory of electromagnetic acceleration for the "separation of currents" has not been adequately developed. Optimum parameters and acceleration regimes of devices are found in the present paper, and the relations obtained are tested experimentally.

The main restriction on the velocity in electromagnetic acceleration is the heating of the body by the flow of current of density j. We determine the optimum current-density function to give maximum velocity for a known magnetic induction $B(t, x)$ in the acceleration region and a current uniformly distributed over the cross section of the conductor. Neglecting air resistance, the acceleration of a conductor for $\mathbf{B} \perp \mathbf{j}$ is

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$$a = B(t, x) j(t, x) \gamma^{-1} \tag{1.1}$$

where x is the displacement, and γ is the density of the conductor material.

Neglecting heat transfer and processes at the seat of the current supply to the accelerated conductor we have

$$\rho = \rho_0 + \frac{k}{\gamma} \int_0^t j^2 \rho dt, \quad k \approx \frac{\rho_0 \theta}{c} \tag{1.2}$$

where ρ is the specific resistance, ρ_0 is the initial value of the specific resistance, θ is the temperature coefficient of resistance, and c is the specific heat.

By differentiating (1.2) we obtain a relation between the current density and the specific resistance of the conductor:

$$j = \left[\frac{\gamma}{k} \frac{d}{dt} (\ln \rho)\right]^{1/2} \tag{1.3}$$

From (1.1) and (1.3) we find an expression for the velocity

$$v(\tau) = \frac{1}{\sqrt{k\gamma}} \int_0^\tau B(t, x) \sqrt{\dot{y}}\, dt \tag{1.4}$$

where τ is the acceleration time, $y = \ln \rho$, and $y = dy/dt$.

The problem reduces to finding the maximum of the functional Φ (1.4).

We distinguish two cases.

1. The acceleration time τ is known:

$$\Phi = \int_0^\tau F_1(t, x, \dot{y})\, dt, \quad F_1(t, x, \dot{y}) = B(t, x) \sqrt{\dot{y}} \tag{1.5}$$

the boundary conditions for y are

$$t = 0, \ y(0) = \ln \rho_0; \ t = \tau, \ y(\tau) = \ln \rho_1$$

and the relation between x and t is determined by the equation of motion.

Euler's equation [6] has the form

$$\frac{d}{dt}\left(\frac{\partial F}{\partial \dot{y}}\right) = 0 \tag{1.6}$$

After transforming (1.6) we obtain

$$\dot{y} = \frac{B^2}{4C_1^2}, \quad j = \sqrt{\frac{\gamma}{k}} \frac{B}{2C_1} = K'B \tag{1.7}$$

The constant K' is determined from the boundary conditions, taking account of the equation of motion.

2. The length of the acceleration region s is known:

By considering t as a function of x we obtain the integrand in (1.5) in the form

$$F_2 [t(x), x, y'(x) / t'(x)]\, t'(x), \quad \Phi = \int_0^s B \sqrt{y' t'}\, dx$$

where primes denote differentiation with respect to x.

The integrand obtained is homogeneous in y' and t'. To find the relation between the extremum of the current-density function and the magnetic induction of the accelerating field we consider Euler's equation [6]

$$\frac{\partial F_2}{\partial y} - \frac{d}{dx}\left(\frac{\partial F_2}{\partial y'}\right) = 0$$

Since F_2 does not contain y,

$$\partial F_2 / \partial y = 0, \qquad \partial F_2 / \partial y' = C_2 \tag{1.8}$$

where C_2 is a constant.

After transforming (1.8) we obtain

$$\sqrt{y' / t'} = B / 2C_2 \tag{1.9}$$

From (1.3) and (1.9) we have

$$j = \sqrt{\gamma / k}\,(B / 2C_2) = K''B$$

The constant K" is determined after integrating (1.9) and using the t(x) relation. Thus, in both cases the optimum occurs when the current density is proportional to the magnetic induction of the accelerating field:

$$j = KB \tag{1.10}$$

Equation (1.10), obtained for an arbitrary time and spatial dependence of the magnetic induction of the accelerating field by starting from the requirement of a minimum heating of the conductor, is general for all optimum electromagnetic accelerators in which the high-velocity limit is set by the heating of the accelerated conductor. Consequently, reaching the highest velocities requires that the current density in the accelerated conductor follow the shape of the field, as is ensured by the constant coefficient K for any specific acceleration process. The maximum value of the current (current density) is determined by the admissible heating, which is taken into account by the value of the coefficient K. For a change in the acceleration conditions [the induction B(x, t), the acceleration path and time] the value of K may also change. It is important that in a specific acceleration process $B(t)/j(t) = \text{const}$. For a given acceleration time the current density does not depend on the maximum value of the magnetic induction.

Equation (1.10) must be satisfied for the optimum operation of high-velocity projection devices.

To find the optimum current-density function for an arbitrary time and spatial dependence of the magnetic field a system of nonlinear differential equations must be integrated, and this can only be done numerically. Expressions can be derived for the current density, the velocity $v(\tau)$, and the acceleration path s for a uniform magnetic field depending only on time. For the practically important case of a magnetic field produced in the discharge of a capacitive energy accumulator

$$B = B_0 e^{-\beta(t+\varphi/\omega)} \sin(\omega t + \varphi)$$

we obtain after transformations

$$j = 2\{k^{-1}\gamma [\ln \rho_1 / \rho_0] [q(\tau) - q(0)]^{-1}\}^{1/2} e^{-\beta(t+\varphi/\omega)} \sin(\omega t + \varphi) \tag{1.11}$$

$$v(\tau) = \frac{B_0 e^{-\beta\varphi/\omega}}{2} \left\{\frac{\ln \rho_1 / \rho_0}{k\gamma} [q(\tau) - q(0)]\right\}^{1/2} \tag{1.12}$$

$$s = \frac{B_0 e^{-\beta\varphi/\omega}}{2k\gamma} \left\{\frac{\ln \rho_1 / \rho_0}{q(\tau) - q(0)} [Q(\tau) - Q(0) - \tau q(0)]\right\}^{1/2}$$

$$q(t) = e^{-2\beta t} \left[\frac{\beta \cos 2(\omega t + \varphi) - \omega \sin 2(\omega t + \varphi)}{\omega^2 + \beta^2} - \frac{1}{\beta}\right] \tag{1.13}$$

$$Q(t) = \frac{e^{-2\beta t}}{2} \left[\frac{1}{\beta^2} - \frac{(\beta^2 - \omega^2)\cos 2(\omega t + \varphi) - 2\omega\beta \sin 2(\omega t + \varphi)}{\omega^2 + \beta^2}\right]$$

where β is the damping factor, φ is the initial angle of acceleration, and τ is the acceleration time.

2. As a consequence of the small acceleration region ($\sim 10^{-2}$ m) and the high average velocity of the conductor, the acceleration time is generally shorter than a half period of the current oscillations in the electric circuit of the accelerator. With sufficient accuracy we can set $\beta = 0$ in Eqs. (1.11)-(1.13) and take account of the damping by decreasing the amplitude of the magnetic induction appropriately. Then for the velocity and the acceleration path we obtain from (1.12) and (1.13)

$$v / v_1 = [2\psi - \sin(2\psi + 2\varphi) + \sin 2\varphi]^{1/2} \tag{2.1}$$

$$s / s_1 = {}^1/_2 [2\psi^2 - \cos 2\varphi + \cos(2\psi + 2\varphi) + 2\psi \sin 2\varphi]\,[2\psi - \sin(2\psi + 2\varphi) + \sin 2\varphi]^{-1/2}$$

$$v_1 = \frac{B_0}{2}\sqrt{\frac{\ln \rho_1 / \rho_0}{\omega k\gamma}}, \quad s_1 = \frac{v_1}{\omega}, \quad \psi = \omega\tau \tag{2.2}$$

TABLE 1

	c, 10^3 J/kg·deg	ρ_0, 10^{-8} Ω·m	γ, 10^3 kg/m³	θ, 10^{-4} deg^{-1}	k, 10^{-18} Hg·Ω·mJ	$\ln \rho_1/\rho_0$	$\frac{\ln \rho_1/\rho_0}{k\gamma}$, 10^{10}
Be	1.79	2.8	1.8	90	1.41	2.54	1.01
Al	0.88	2.45	2.7	45	1.25	1.4	0.42
Cu	0.38	1.56	8.96	43	1.9	1.85	0.11
Fe	0.65	8.9	7.9	65	9.2	9.65	0.034
Constan-tan	0.42	50.0	8.9	10^{-2}	$1.2 \cdot 10^{-2}$	$5 \cdot 10^{-4}$	0.0047

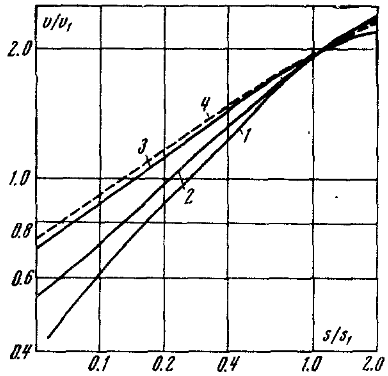

Fig. 1

Using (2.1) we find the optimum angle of the initial acceleration φ_0 for which the velocity is maximum:

$$\varphi_0 = n\pi / 2 - \psi / 2, \ \psi / \pi < n \leqslant 1 + \psi / \pi$$

For $\varphi = \varphi_0$ we find from (2.1) and (2.2):

$$v_0 = v_1 \left[2 \left(\psi + |\sin \psi|\right)\right]^{1/2} \tag{2.3}$$

$$s_0 = v_0 \psi / 2\omega \tag{2.4}$$

From Eqs. (2.3) and (2.4) we find expressions for the required magnetic induction B_0 and the energy W_0 stored in the accumulator

$$B_0^2 = \frac{k\gamma}{\ln \rho_1 / \rho_0} 2\omega v_0^2 \left[\frac{2\omega s_0}{v_0} + \left|\sin \frac{2\omega s_0}{v_0}\right|\right]^{-1} \tag{2.5}$$

$$W_0 = \Delta s \omega k \gamma v_0^2 \left[\mu_0 K_1 K_2 K_3^2 \ln \rho_1 / \rho_0 \left(\frac{2\omega s_0}{v_0} + \left|\sin \frac{2\omega s_0}{v_0}\right|\right)\right]^{-1} \tag{2.6}$$

where Δs is the size of the acceleration region.

The coefficients K_1, K_2, and K_3 have the following physical meaning: $K_1 = L_1/(L_0 + L_1)$ characterizes the fraction of the total energy stored in the accumulator which is transferred to the magnetic field of the coil, where L_1 is the inductance of the coil, and L_0 the self-inductance of the accumulator:

$$K_2 = \exp\left\{-\frac{r \operatorname{arc\,tg} 2r / [\omega (L_1 + L_0)]}{\omega (L_1 + L_0)}\right\}$$

takes account of the damping of the discharge current up to the time of the first maximum as a result of Joule losses in the resistances (r) of the discharge circuit; $K_3 = B_0/B_m$ characterizes the decrease of the field in the acceleration region due to leakage.

Equations (2.3)-(2.6) were found by assuming a constant acceleration time. More frequently the length of the acceleration path is kept constant for various acceleration regimes. In this case the determination of the optimum angle of the initial acceleration is reduced to solving a system of transcendental equations, which can only be done numerically. Figure 1 shows the velocity as a function of the acceleration path for various angles of the initial acceleration [φ = 1) 0; 2) 30°; 3) 60°; 4) φ_0]. Analysis of the curves shows that over a wide range of parameters $0.02 < s/s_1 < 1.0$ the relations found permit the calculation of the optimum parameters of the device and the acceleration regime.

Equations (2.1) and (2.2) show that the best material for acceleration in a pulsed magnetic field is one for which $\ln(\rho/\rho_0)/k\gamma$ is maximum. Table 1 lists data for a number of materials. Calculation shows that the velocity can be doubled by replacing a copper conductor by aluminum, and increased 50% by replacing it with aluminum–beryllium.

The criterion obtained can be expressed in terms of the integral of the square of the current density ("current integral") [7].

$$J = \int_0^t j^2 dt$$

It follows from (1.2) that

$$\ln \rho_1 / \rho_0 = \frac{k}{\gamma} \int_0^t j^2 dt = \frac{k}{\gamma} J$$

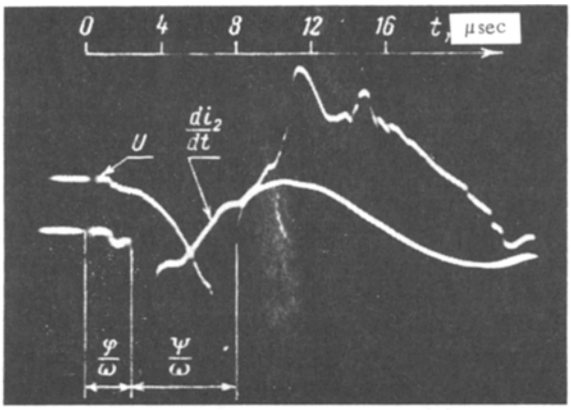

Fig. 2

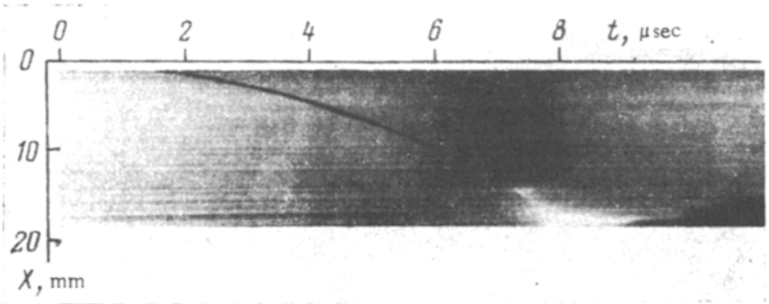

Fig. 3

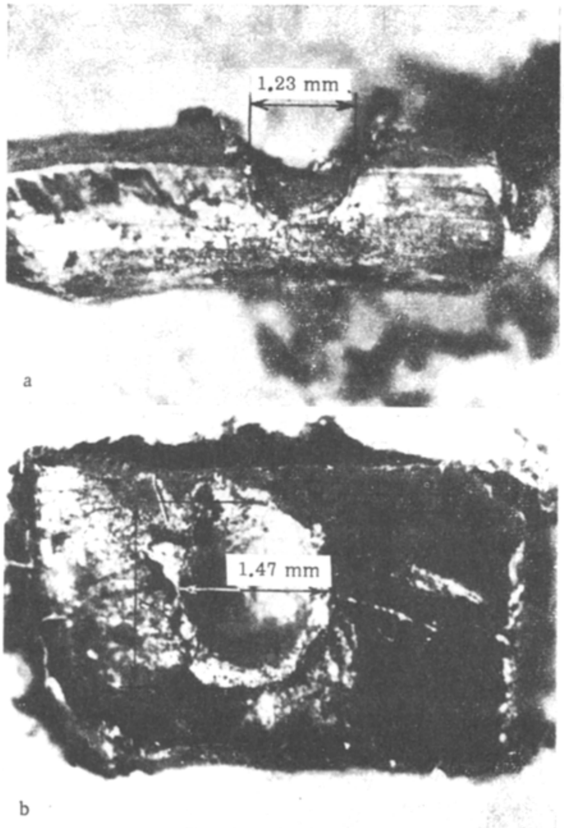

Fig. 4

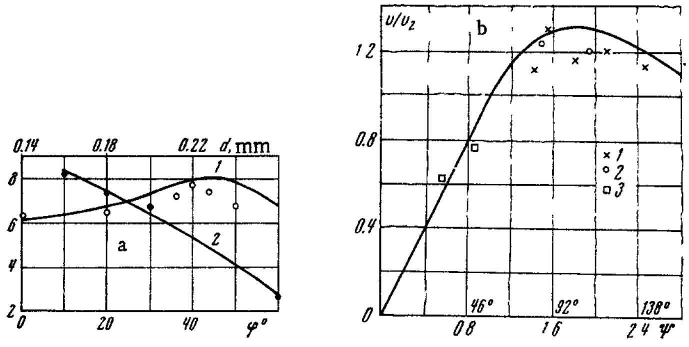

Fig. 5

For the criterion characterizing the effectiveness of the acceleration of a given metal in a pulsed magnetic field we obtain

$$(k\gamma)^{-1} \ln \rho_1 / \rho_0 = \gamma^{-2} J$$

The best material for projection is one for which the criterion is largest.

In [7] the limiting velocity which can be communicated to a "thin sheet" whose thickness d is small in comparison with the thickness of the skin layer is found to be

$$v = {}^1/_2 \mu d \gamma^{-1} J$$

This relation does not characterize solely the material properties of the accelerated conductor, but also the limitations of the method of acceleration, since it involves the thickness of the sheet, as well as the material properties. The main drawback of this acceleration scheme is the assumption of the simple rigid relation between the field H and the current in the accelerated conductor:

$$jd = i = -\bar{H}$$

In the present scheme the velocity limit set by the heating of the conductor can be raised for an independent change in the relation between the field and the current density. Assuming, for example, that $H = njd$, we find that the limiting velocity increases by a factor of n^2; i.e., the theoretical limitation on the limiting velocity is removed.

3. To test the relations obtained and to investigate the dynamics of the acceleration a series of experiments was performed on an induction accelerator with an auxiliary transformer, permitting separate control of the magnetic induction of the accelerating field and the current in the conductor. The accumulator, built of six IMU-5-140 capacitors, had a nominal P. D. of 5 kV, an energy content of 11 kJ, and a natural frequency of 46 kHz. A three-electrode air spark gap with coaxial electrodes was attached to the lead of each capacitor, and the spark gaps were connected through flat busbars to the collecting electrode on which the accelerator was mounted. The conductor to be accelerated was connected to the secondary winding of a high-Q air-core transformer whose primary winding carried the current from the capacitive energy accumulator. The angle of initial acceleration was controlled by a device using the rupture of a foil connected in parallel with the conductor to be accelerated to turn on the current. In the experiments described the maximum amplitude of the current through the coil (i_1) was 850 kA, and that in the accelerated conductor (i_2) 2.5 kA. Arcing was decreased and the contacts of the accelerated conductor were improved by using converging current-carrying rails with a vertex angle ≈15°.

The velocity of the body was measured in two ways: by high-speed photography of the acceleration process using a shadow method, and by an oscillogram of the voltage across the contact rails. The voltage across the rails is

$$U = e_1 + e_2 + u, \quad e_1 = Bvl, \quad e_2 = M\, di_1 / dt,$$
$$u = d(i_2 L_2) / dt + i_2 r \qquad (3.1)$$

where e_1 is the emf due to the motion of the accelerated conductor (V), B is the magnetic induction of the accelerating field (Wb/m^2), v is the velocity of the conductor (m/sec), l is its length (m), e_2 is the emf due to mutual inductance (V), M is the mutual inductance (H), and u is the potential drop across the rails and the accelerated conductor due to the current i_2.

We estimate the maximum values of the terms in Eq. (3.1). For $B = 50$ Wb/m^2, $v = 8 \times 10^3$ m/sec, $l = 4 \times 10^{-3}$ m, $e_1 = 1600$ V. The maximum value of the mutual inductance calculated as in [8] is 6×10^{-9} H. Thus, $e_2 = 1300$ V. Since $i_2 \ll i_1$, $u \ll e_2$. It follows from these estimates that at the instant of impact with the target a sharp decrease in the voltage across the rails must be observed, from which it is impossible to determine the instant at which acceleration stops.

Figure 2 shows oscillograms of the voltage across the rails and the derivatives of the current for an experiment with $B = 50$ Wb/m^2, $s = 16 \times 10^{-3}$ m, $l = 4 \times 10^{-3}$ m, and the angle of initial acceleration 12°. The measured acceleration time was 7 μsec and the calculated velocity 7×10^3 m/sec. The high-speed photographs of the acceleration process for the same experiment are shown in Fig. 3. The optical system of an OI-2 illuminator with a spark light source was used for lighting. It can be seen from Fig. 3 that for a velocity $v \approx 3$ km/sec the air in front of the projected body glows. As the velocity increases dense air jets flowing around the body shield it; the velocity was measured with respect to the boundary of the disturbance. The values of the velocity determined from the oscillograms and from the high-speed photographs agree within the limits of experimental error (~10%).

The shape of the body was monitored by x-ray photographs of the acceleration process. The radiation source was a sharp-focused "open" three-electrode x-ray tube [9]. The density of blackening of the image of the conductor was measured with a type MF-4 microphotometer. No significant change in density was observed along the length of the accelerated body, showing that it remains whole and maintains its diameter and orientation and exhibits no inhomogeneities or breaks along its length. During acceleration a conductor unavoidably becomes shorter as a result of erosion at the contacts.

In the experiments the target was placed either in the coil (inductor) or outside it. In the latter case there was an opening in the coil along which the body could change its orientation or shape. When the target was placed in the coil itself an elongated crater was obtained, since the length of the accelerated conductor (2-4 mm) was much larger than its diameter (0.1-0.2 mm). Figure 4a shows a side view of such an elongated crater. In the experiment (Fig. 4b) the target was fastened inside the coil with an insulating sleeve partially covering the active surface; this accounts for the shape of the crater in Fig. 4b.

For comparison experiments were performed on the acceleration of 0.08-mm-diameter conductors [$j = 4j_*$, where j_* is the value of the current density calculated by Eq. (1.11)]. In this case approximately 1.4 μsec after the beginning of acceleration an electric explosion of the conductor was observed. The instant of explosion was determined from the oscillogram of the derivative of the current in the conductor and by the high-speed photographs; from then on the plasma was accelerated. The nature of the glow observed in the high-speed photographs differs significantly from the case of the acceleration of a solid conductor, and the final velocity in air did not exceed 4 km/sec.

The results of the bombardment of aluminum and copper targets are shown in Fig. 4; a) aluminum target, $v = 6.8$ km/sec (side view); b) copper target, $v = 10.5$ km/sec (top view). The presence of craters and chips of the rear surface, characteristic of high-velocity impact, are clearly visible.

The dependence of the velocity of aluminum conductors on diameter (curve 2 for a constant angle of the initial acceleration $\varphi = 40°$) and on the angle of initial acceleration (curve 1 for a constant diameter $d = 0.17$ mm) are shown in Fig. 5a ($B = 50$ Wb/m^2, $l = 4$ mm, $s = 17$ mm; the calculated curve is plotted as a solid line). We introduce the reference velocity $v_2 = (B_0 j_0 \gamma^{-1} s)^{1/2}$, where B_0 and j_0 are, respectively, the maximum values of the induction and the current density. Then in relative units the experimental results can be represented in the form of a generalized relation $v/v_2 = f(\psi)$ on Fig. 5b; 1) $B = 50$ Wb/m^2, $s = 17$ mm, $\varphi = 0$; 2) $B = 80$ Wb/m^2, $s = 18$ mm, $\varphi = 0$; 3) $B = 50$ Wb/m^2, $s = 13$ mm, $\varphi = 0$; $\psi = \omega\tau$ is the angle of acceleration, and the calculated curve is plotted as a solid line. The largest experimental velocity of 10.5 km/sec was obtained by accelerating a piece of aluminum wire 4 mm long and 0.16 mm in diameter with $m = 2 \times 10^{-7}$ kg, $B = 80$ Wb/m^2, $s = 18$ mm, $\varphi = 34°$. The experimental results and the calculations are in good agreement, showing the reliability of the relations obtained and the possibility of a further increase in velocity.

LITERATURE CITED

1. R. J. Eichelberger and J. H. Kineke, "Hypervelocity impact," in: High-Speed Physics, Springer-Verlag, New York (1967).
2. A. J. Cable, "Hypervelocity acceleration," in: High-Velocity Impact Phenomena, Academic Press (1970).
3. V. N. Bondaletov, "Induction acceleration of conductors," Zh. Tekh. Fiz., 37, 280 (1967).

4. V. N. Bondaletov and G. M. Goncharenko, "Acceleration of conductors in a pulsed magnetic field," Zh. Tekh. Fiz., 40, 2209 (1970).
5. K. Thom and J. Norwood, "Hypervelocity electromagnetic mass accelerator," in: Symposium on Hypervelocity Techniques, Inst. Aero. Sciences, New York (1960).
6. L. É. El'sgol'ts, Differential Equations and the Calculus of Variations [in Russian], Nauka, Moscow (1969).
7. H. Knoepfel, Pulsed High Magnetic Fields, North-Holland, Amsterdam (1970).
8. P. L. Kalantarov and L. A. Tseitlin, Calculation of Inductances. Reference Book [in Russian], Énergiya, Leningrad (1970).
9. B. N. Zolotykh and V. N. Sidorov, "Sectional sharp-focusing pulsed x-ray tube," in: Problems of the Electrical Treatment of Materials [in Russian], Akad. Nauk SSSR, Moscow (1962).